

Review on Geotechnical Centrifuge Modeling Theory

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Abstract: Stresses due to self weight by placing model in centrifuge role behavior have been used extensively in various fields of engineering. In certain prototypes, should be reproduced in the model. Also report will cover the principles, advantages, and problems of centrifuge testing with the discussion of use of scaled model tests, Hence the relationships between the model and prototype, will be obtain. It will examine the various uses of centrifuge has been put, for modelling soil, rock and ice behaviour, with studying gravity tectonics.

Keywords: Centrifuge, Scale, Prototype, Model, Stress.

I. INTRODUCTION

The basic theory on which centrifuge modeling is based will be the principle of stresses at geometrically similar points for prototype and model will be the same.

To produce at corresponding points in a small scale model, the same unit stresses

that exist in a full scale structure, the weight of the material of the model must be increased in the same ratio that the scale of the model is decreased with structure. The increase in weight can be obtained by the use of centrifugal force, for that model being placed in a revolving apparatus.

If the model and prototype are made of materials with identical mechanical properties, then the strains in the model and prototype will also be identical. We found a-L scale model of a prototype is spun at N_g on the centrifuge, again the model's behaviour is thought to be similar to the prototype's behaviour. Therefore theory is valid for the following assumptions:

- (a) The model is a correctly scaled version of the prototype;
- (b) The scaled model also subject to an ideal N_a gravity field, behaves like the prototype at I_g
- (c) The centrifuge produces in ideal gravitational field. Therefore assumptions will be examined in detail of the following sections.

A. The model is correctly scaled in the version of prototype

This assumption is an exactly scaled version of the prototype, requires scaling relations between the model and prototype be satisfied. Such scaling relationships are obtained from a dimensional analysis of the relevant variables, or from consideration of the governing equation. The details of the two scaling approaches will be discussed with a summary is provided below:

The suffixes 'p' and 'm' stand for prototype and model respectively, and L_p and L_m represent the length of the prototype and the length of the model respectively, the relation obtained as

$$\frac{L_p}{L_m} = N \quad (1.1)$$

Scaling relation and 'N' is the scale factor. Similarly scaling relations can be established for other properties i.e.: unit weight, velocity, acceleration, etc.

In many cases however, exact similitude between model and prototype is not possible. For that, variables whose effects are known and influence the behaviour, are allowed to deviate from their scaled values.

Scaling down a prototype, especially with large scale factors, may result in a loss of prototype detail. Which may not be of importance, sometimes, it may be crucial, centrifuge modelling of gravity tectonics processes, the value of the scale factor 'N' is of the order of 104 [Ramberg (1965)]. In such situation, modelling of a rock layer a few feet thick is not possible. However, the structures being modelled have dimensions in miles. In other cases, however, this may not be true. For example, In prototype the behaviour being studied, and correctly scaled included in the model. Scaling down a prototype may also result in parameters,

which do not have an effect on prototype behaviour, significantly influencing model behaviour.

These effects are known as scale effects scaling done to the prototype. As using the centrifuge, the soil material is not scaled in grain size. In such a case, the model footing may be scaled down to the extent that the individual size of the grains would begin to affect the footing behaviour.

Scale effects occur in all models. Their effects are reduced, by building the model large guard against scale effects is to check is called the 'Internal Consistency' of the experiment The 'Modelling of Models' technique. This technique consists of modelling the same prototype at different scales. So, if the results of a **1/100** scale model of a particular prototype, tested at **100g** are the same as those of a **1/20** scale model of the same prototype tested at 20g, then one can conclude that the scale effects are not significant. Examples of parameters that could result in scale effects are grain size and surface roughness.

Apart from loss of prototype detail and scale effects, it may not be possible to model certain prototype characteristics. Like the crystal structure of model ice may differ from that of the arctic sea ice being modelled. So it has been suggested that results from the tests using model ice be modified analytically to account for the difference in ice crystal structure.

B. Scaled model is an Ideal N_g (Gravity field)

Assumption is that the $1/N$ scale model, subject to an ideal N_g gravity field acting on the surface of an N_g planet), behaves like the prototype at **1g**. This assumption requires that, the model material at **17g** has the same material properties as the model

material scaling is done on the basis of the material properties at **1g**. Phenomena occur in the prototype at **1g** occur in the model at **N_g**.

C. The centrifuge Produces in Ideal gravitational field

The centrifuge produces an ideal **N_g** gravitational field. An ideal gravitational field is considered to be the gravitational field acting on the surface of an **N_g** planet. The earth can be considered an **N_g** planet, where **N=1**. Any mass resting on the surface of such a planet is subject to two forces that give it its weight. One force is the centrifugal force due to rotation of the planet about its axis while the other is due to gravity and is given by Newton's law of universal gravity. Thus the force acting on a body on the surface of a planet is given as

$$F = \frac{Gm_1 m_2}{r^2} - m_1 \omega^2 r$$

(1.2) Where F= Force acting on the mass resting on or in the earth

m_1 = Mass of the body on or in the earth

m_2 = Mass of the earth

r = The radius of the earth at that point

ω = Angular velocity of rotation of the earth

G = The universal gravitational constant.

In the quantity $\frac{Gm_1 m_2}{r^2}$, the value $m_1 \omega^2 r$ is so small that it can be neglected. The gravitational field on a planet is not a constant, but varies with distance from the center of the earth, up to depths in geotechnical engineering, this change is so small that it can be neglected. In civil engineering, the acceleration due to gravity

is considered constant, throughout the prototype.

The direction of the gravity field changes on the surface it acts towards the earth's center. In civil engineering projects, this change in direction is too small. Hence, simulation of an ideal gravity field requires that the **g** level does not change in both magnitude and direction at all points within the model.

The centrifuge, does not provide an exact replication of the above conditions, comes very close to doing so. It is discussed in the following paragraphs.

Let a mass '**M**' be placed on the arm of the centrifuge at a distance of '**R**' meters from the axis of rotation, and spun at an angular velocity of **W** rad/sec. There are two ways of looking at this body as it rotates in the centrifuge. One way is from a fixed external frame of reference, other is from a reference frame that rotates with the body. Any force that occurs in the rotating frame but not in the fixed frame is a pseudo force.

Examples of pseudo forces are the centrifugal force and the coriolis force, both of which are details as below:-

The tangential velocity of the mass as it rotates about the axis is given by ωR . If the mass were unrestrained, it would shoot out with this velocity along a path tangential to the circumference of rotation. Viewed externally, the mass is pulled out from this straight line and around a curve of radius '**R**' meters. When the rotation is uniform, it can be shown that there is an acceleration $A = \omega^2 R$ that is always directed radially inward. This acceleration is called the centripetal acceleration.

This centripetal acceleration acting on the rotating mass results in a force called the centripetal force. Since this force acts in a fixed frame, it is a real force. When the reference frame rotates with the mass on the centrifuge, there will be a force acting radially outwards that is equal in magnitude to the centripetal force, but opposite in direction. This is the centrifugal force, and since it exists only in a moving reference frame, it is a pseudo force.

As explained earlier, ideal conditions in an artificial g field require that the acceleration does not change in magnitude and direction at any point in the model. Now effect of model depth will be as consider a model rotating at a distance 'R' from the center of rotation, at an angular velocity of ω rad/sec. All points in the model are not equidistant from the axis of rotation and consequently, the acceleration field in the rotating model increases linearly with depth.

At the upper surface, both the model and prototype have zero total stress. Now consider a point at a distance R from the axis of rotation, and at a depth 'aR' below the surface of the model.

The acceleration at this point is $\omega^2 R = Ng$ and the scale factor at this point is N. For the prototype, this point would correspond to a point at a depth NaR. The vertical stress at this point in the prototype would be

$$\sigma_v = \rho g Na R \quad (1.3)$$

where ρ is the mass density of the prototype material. For the model in the centrifuge, however, the vertical stress at this point is given by an integration to the depth 'aR', since the acceleration field from R-aR = R (1-a) (the radius to the ground surface) to R is not constant.

Consequently, the stress at this point in the model is given by

$$\begin{aligned} \sigma_v &= \int_{R(1-a)}^R \rho \omega^2 r \, dr \\ &= \frac{\rho \omega^2 a R^2 (2-a)}{2} \quad (1.4) \end{aligned}$$

Equating 1.3 and 1.4

$$\begin{aligned} \rho g Na R &= \frac{\rho \omega^2 a R^2 (2-a)}{2} \\ \frac{Ng}{\omega^2 R} &= \frac{2-a}{2} \quad (1.5) \end{aligned}$$

Now, in the model at depths less than aR, there will be an understress in the model as compared to the prototype, and for points at depths greater than aR, there will be an overstress compared to the prototype. The depth below the model surface they intersect at depth aR and is the only depth in the model where the model stresses are exactly equal to the prototype stresses.

II. ADVANTAGES OF CENTRIFUGE TESTING

- 1) The advantages over modelling at **1g**, for which gravity is a dominant factor in prototype behavior is found as:-
- 2) Modeling at Ig does not produce the same stresses in the model and prototype and geotechnical materials are non-linear, so the strains at these stress levels are not the same as those of the prototype. Modeling at Ig does not results in good model-prototype.
- 3) The basic principle of centrifuge modeling is that the stresses at various points in the model are approximately the same as in the

prototype. i.e. if the same materials are used in the prototype, then the strains will be the same, which increased model-prototype. That techniques to measure the material properties at low stresses need not be developed. The stresses imposed on the materials are the actual prototype stresses. These are some of the major advantages of centrifuge testing. Model studies using the centrifuge, hence demonstrate behavior is close to prototype behavior. These test data can be used to validate existing theoretical models. therefore the model is verified on the basis of centrifuge tests, and can be used to run parametric studies of the prototype.

- 4) If no theoretical model exists, then the data from the tests may provide added insight into the phenomenon, which leads ultimately to the development of a theoretical model. Also provide realistic data for the verification/development of theoretical models is an important advantage of the centrifuge.
- 5) With all above reasons, the centrifuge has been used extensively for the verification of theories, for the study of deformation and failure of problems involving the flow of water, as well as for various dynamic problems such as pile driving, earthquake effects, etc.
- 6) Direct modeling of a particular prototype is sometimes suggested but, more research and development must be done before to be successfully accomplished. With the reasons cited above, the centrifuge

has been used to study a wide variety of problems in soil mechanics, rock mechanics, ice mechanics and gravity tectonics.

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