A Fair Game Theoretic Approach for Spectrum Sharing in Cognitive Radio Networks

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Abstract—In this paper, we propose a model for spectrum leasing in cognitive radio networks (CRN) with the help of game theoretic approach. This model can efficiently utilize the spectrum and limits the interference within the Pareto optimal boundary (POB). In this model, primary user access point (PUAP) distributes the licensed bandwidth to multiple secondary users (SUs) *via* auctioning. Secondary user (SU) transmits over the allocated sub-carrier for fixed duration. After that duration, PUAP re-allocates the sub-carrier to another SU. This model can reduce the total transmission time of SUs and maximizes the pay-off of PUAP (number of SUs) by implementing the Stackelbergs competition model for next auction. The proposed approach can reduce the interference caused by the transmitting nodes and provides better options for the SUs to maximize their pay-off function (information rate). In this model a new form of hybrid strategy is implemented between PUAP and SUs. Here, PUAP controls the spectrum leasing process and able to switch the strategies dynamically from non-cooperative to cooperative. This hybrid strategy is more robust and efficient compared to the conventional strategies of game theory.

The proposed hybrid strategy, secures the information of users from any type of vulnerability and at the same time provides maximum possible pay-off to each user. Hybrid game approach will help the users to utilize the available resources efficiently. However, this model can be implemented if SUs and PUAP are computationally efficient.

Keywords—Cognitive Radio Network (CRN), Game Theory, Pareto Optimal Boundary (POB)

I. Introduction

As the demand of high data rate is increasing, bandwidth is going to be the most valuable asset. Efficient bandwidth utilization is always been a problem for wireless communication. Cognitive radio networks (CRN) came out to be a promising solution for bandwidth utilization [1].

In the present era, most of the wireless networks use fixed spectrum allocation policies [2]. These spectrums are called licensed spectrum. Demands of the users on licensed spectrum are not constant over time and according to survey conducted by FCC $15-85\%$ of spectrum is un-utilized on an average [3]. The model aims to propose the most efficient form of concurrent communications of cognitive users, competing over the available spectrum provided by primary users (PUs). Global optimization based algorithms do not have control on amount of interference generated by the transmitters [1]. However, interference offered by secondary users (SUs) to PU should be kept below some threshold level to increase the pay-off of users. Further, global optimization based algorithms are computationally complex.

Game theory is an area of applied mathematics that deals with interactive decisions. Game theoretical approaches have been used to model many communication problems like power control and resource sharing. In the CRN, game theory is widely accepted for spectrum leasing [4]. There are two general approaches of game theory, cooperative and non-cooperative. For cooperative game theory, players have to share their information with the other players through a central node, which is having information related to all the players. Even if one of the players is vulnerable, then the issue of information security arises. Cooperative game strategy is a time consuming process [4]. However, this type of game strategy helps SU to maximize their pay-off function. Another game approach is non-cooperative game. These types of games are faster and secure, because players need not share their strategy or any type of information, so there is no need of centralized authority. Non-cooperative approach reduces the chances of maximizing pay-off function [5] because they don't have knowledge about Pareto optimal boundary (POB) [6], if they play rational.

A. Related work and Paper organization

There are lot of literature available for game theory and auction theory which is used in CRN for various purposes. Haykin [7] provides the underline structure of CRN. Chen [8] used hybrid game strategy to maximize the pay-off function of SU. Here, users are hybrid and SU has freedom of choosing PU band for transmission. PUs will provide details, like bandwidth and target bit rate, but there is no consideration for the interference provided by the SU to PU.

Stanojev [9] modeled a cooperative game scenario between users and relays. Model implements the auction theory to select the preferable relay for sending users information. In this method, a slot is re-transmitted over the allocated spectrum by the relay, relay has to forward the data of the user as well as obtain an opportunity to send his own data. This technique resolves the problem of both user and relay, but relay may be vulnerable. Cooperative game strategy used by [10] for spectrum sharing. In this system, both players communicate via centralized authority and find out some key factors for transmission like interference cap. This type of strategy helps player to get maximum pay-off, but comes with some limitation of information security and speed of gaming. Non-cooperative game strategies are used for spectrum leasing

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by [11], [12]. In [11], author considered spectrum releasing as an Oligopoly market competition. Cournot game is used to solve this problem and Nash equilibrium (NE) [13] is the solution. In [12], author considered iterative prisoner's dilemma (IPD) for spectrum releasing and used various game strategies for finding equilibrium point. Iterative game is used to remove the dominance of the dominant strategy solution (DSS) is used by [14], [15].

Simeone *et al.* [16] proposes the cooperative model between the users for the spectrum sharing. In [17] author, solve the issue of spectrum sharing with the help of Stackelberg game approach. Li *et al.* [18] proposes a model of spectrum leasing by PUs. The model is based on the coalitional strategy and differ from the [16], [17] only on the aspect of pricing, of the spectrum. The model proposed in this paper is different from the model proposed by Li and others, in the following aspects.

- It provides a systematic solution for spectrum leasing *via* auctioning.
- Hybrid model is used by PUAP to lease the spectrum.
- Markov process is used to remove the vulnerability of Vickery auction technique.
- PUAP limits the interference provided by SUs, using hybrid and Markov process.

Hybrid approach efficiently fills the gaps provided by previous approaches. In the proposed hybrid strategy PUAP can change strategies dynamically, from non-cooperative to cooperative. Behavior of PUAP depends on the strategy of SU. PUAP works adaptively, if SU play fair then the PUAP change strategy from non-cooperative to cooperative otherwise PUAP will terminate the agreement. The hybrid strategy provides chance for SUs to improve the pay-off function without interfering with the POB. Application of the Markov chain is also proposed to remove the vulnerability of the Vickery auction mechanism.

In this paper, II covers the overview of the system model. Strategy followed by SUs to get auction is discussed in III, it also includes proposed generalized form of *tit for tat* approach capable of handling multiple users. Hybrid approach is proposed in IV for spectrum leasing. Re-allocation strategy of sub-carriers is discussed in V. Numerical results are discussed in VI and concluding remarks are provided in VII.

II. System Model

Auction scenario is illustrated in Fig 1. Here, PU Access point (AP) auctions the un-utilized spectrum and call for the participants. Interested SUs forward their willingness to PUAP. PUAP auctions the sub-carrier one by one and the number of SUs may be variable, it has the auction and strategy server which controls the entire process.

A. Auction Framework

PUAP has un-utilized spectrum. To utilize this spectrum, PUAP goes for spectrum leasing by auctioning technique. PUAP have total L sub-carriers out of which l are un-utilized. So, the necessary condition for spectrum leasing is:

$$L \geq l \geq 0 \tag{1}$$

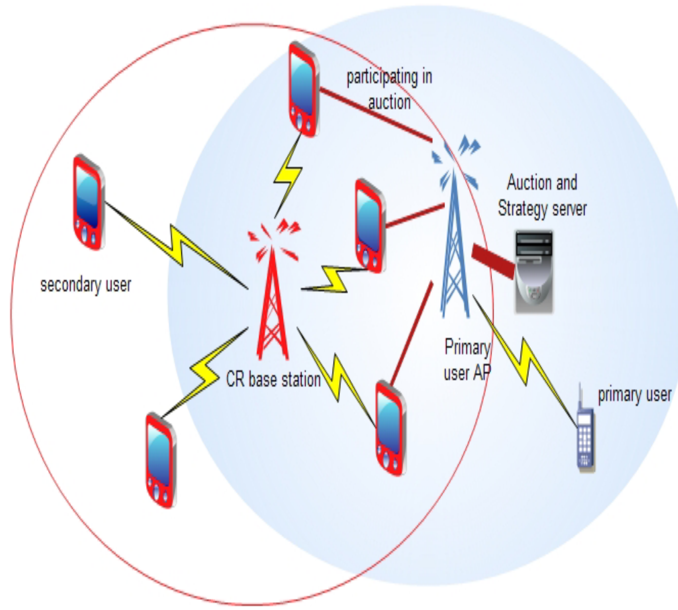

Fig. 1. Scenario for auctioning.

PUAP auctions the l sub-carriers one by one till $l = 0$. Interested SUs take part in the auction process. Initially SUs will prefer that PUAP which has more number of sub-carrier for auction. Then SUs pass on their bids to PUAP. The bid B_j ($j = 1, 2, 3, ..., n$ represent j^{th} user) depends on three parameters R^s_j, D_j, and Pr_j. Where, R^s_j, D_j, and Pr_j represents the reliability evaluated by SU, data to be transmitted, and price for the bandwidth respectively. The reliability to be followed in this paper is:

- R^s_j evaluated by j^{th} SU itself on the basis of the number of re-transmissions needed to successfully transmit a chunk of data between SU to AP (CR base station). As the number of re-transmissions increases reliability of the user decreases [9]. Based on this, the proposed reliability R^s_j is calculated as follows:

$$R^s_j = \frac{(D_j)}{[(t_j) + (e^{re}_j)]C^{o,s}_j} \tag{2}$$

  Here, t_j, e^{re}_j, $C^{o,s}_j$ represent the time taken to transmit data, number of re-transmissions, and average data rate between SU to AP respectively.
  From (2), if $e^{re}_j = 0$, R^s_j is 1. As the value of of e^{re}_j increase, R^s_j tends to decrease.

$$0 \leq R^s_j \leq 1 \tag{3}$$

- R^{sp}_j is the reliability of the j^{th} SU as evaluated by PUAP. The basis of the evaluation is the pay-off achieved by the SU $w.r.t$ the POB. R^{sp}_j can take only two values 0 and 1. R^{sp}_j is set to be 1, If the pay-off of SU is not harming the pay-off of other users otherwise it is set to 0 value.

PUAP adopts Vickery second price auction technique for spectrum leasing. PUAP calls the auction for l sub-carriers one by one, *i.e.*, number of auctions equals to l.
PUAP distributes the auction on the basis of bids provided by

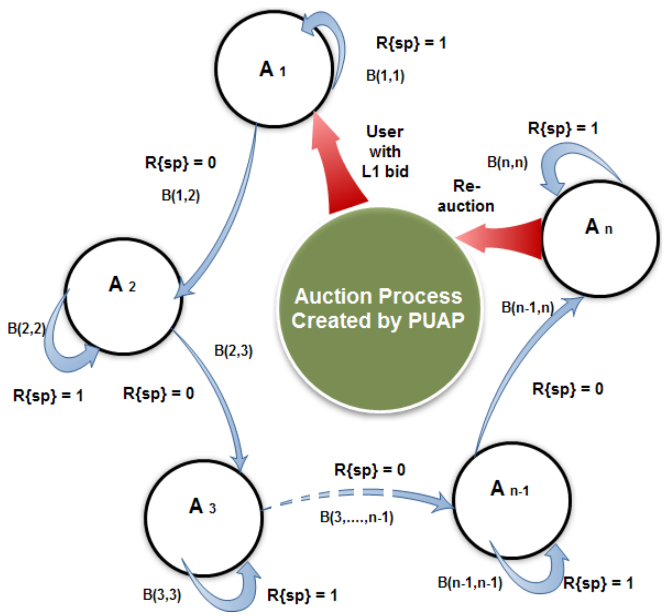

Fig. 2. A conceptual illustration of an auction technique using Markov chain model.

SUs. PUAP will calculate the importance of the bid, T_b:

$$T_j^b = f\{R_j^s, Pr_j, l\} \qquad (4)$$

Major portion of T_j^b depends on the value of R_j^s. The effect of Pr_j is a constraint of l. When unutilized sub-carriers are less PUAP expect high Pr for sub-carriers.

PUAP arranges the bid in descending order (on the basis of T_j^b). Without loss of generality $j = 1$ is the highest bidder A_1 be the SU which is having maximum T^b (T_1^b) which acquires the sub-carrier. After each auction:

$$l = l - 1 \qquad (5)$$

A_1 has to fulfill the cost of second highest bid (T_2^b) using Vickery second price auction technique.

1) System Performance over Vickery technique: Vickery auction provides benefit to PUAP and attracts more SUs. However, SUs may exploit the vulnerability of PUAP. This vulnerability may arise due to fake biding made by SUs. To remove this vulnerability, we propose to use Markov chain model. PUAP will find out the reliability assets R_1^{sp} of the SU. If $R_1^{sp} = 0$, the user is vulnerable in nature and bid shifts to next user A_2 and so on.

The Markovian structure of the process is shown in Fig.2. There are total $n + 1$ states with the initial state represents PUAP calling auction. State A_1 is represented by the SU having $max(T^b)$. A_2 represents the SU having second highest bid and all other states from $A_2, A_3, ..., A_n$ represent the SU with descending value of bid. Transition between the states depends on the value of R_j^{sp}. Before transition, every state holds the sub-carrier for the T duration. This duration is called probation period shown in Fig. 3. During probation period PUAP finds out the R_j^{sp} of SU.

- If $R_1^{sp} = 1$:
  Markov process founds the stable state. SU moves out of the probation period. SU can hold the sub-carrier for t duration.
- If $R_1^{sp} = 0$:
  There is transition of state from A_1 to A_2 and so on.

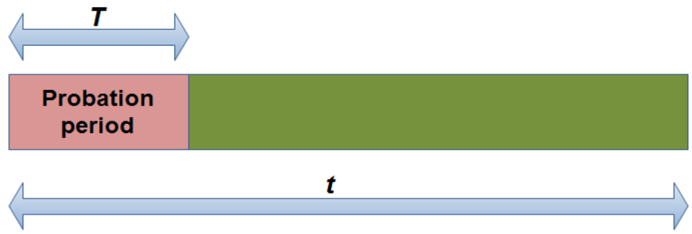

Fig. 3. Total transmission time t and the probation period T.

B. Working concept of model

In the proposed model (Fig. 4), PUAP calls for the auction of unutilized sub-carriers. SUs follow cooperative or non-cooperative strategies for vying of sub-carriers. Here, pay-off of SUs are the sub-carriers. Proposed hybrid strategy is a type of game model based on reputation (GMBR) [8]. Hybrid approach is implemented by PUAP and the pay-off of SUs changes to information rate. After t duration PUAP re-allocate the sub-carrier to other SU, reducing the total time of transmission for SUs. PUAP gains maximum number of SUs for next auction by implementing the Stackelberg's competition model. A further detailed discussion of the approaches is provided in the following sections.

III. Game Strategies Followed by SUs for Auction

SUs follow different game strategies for vying of sub-carriers. Here, we have discussed different strategies inter-linked to each other.

A. Cooperative technique

SUs are the selfish users and always try to increase their pay-off function. For auction, pay-off functions for the SUs are sub-carriers. Desperate SUs make collation with other SUs and bargain on the pay-off function [19].

- Effect of collation increases on auction as the number of SUs increases. Here, n_l SUs are in collation out of n. For collation $n_l - 1$ users forward underappreciated bids so that the probability of belonging A_1 to group increases. Significant increase in n_l also increases the probability of winning sub-carrier significantly.
- Desperate SUs initializes the formation of collation. SUs working for high pay-off function are the desperate SUs. Other SUs joining collation bargain with desperate SUs [5].
  Let, x_h is the number of sub-carriers needed by desperate SUs.
  x_l is the number of sub-carriers needed by other SUs.
  a_h is the number of sub-carriers desperate SUs already have.

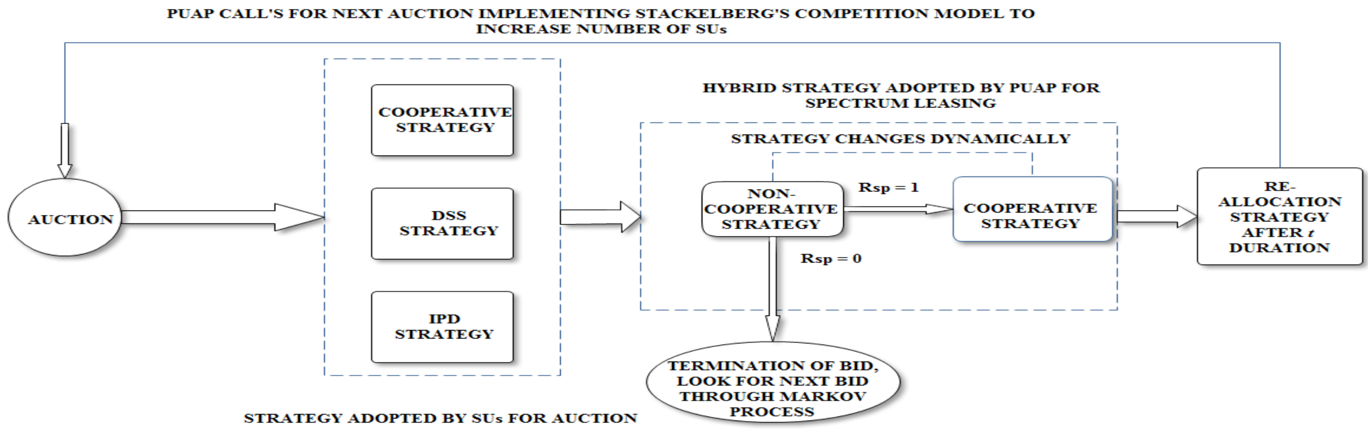

Fig. 4. Flow diagram of the proposed model.

a_l is the number of sub-carriers other SUs already have.

$$x_h > x_l : a_l \geq a_h \quad (6)$$

Then the Pay-off function for bargaining is [4]:

$$u_h^{bargain} = log(a_h + x_h) \quad (7)$$

$$u_l^{bargain} = log(a_l + x_l) \quad (8)$$

And for the case of no bargaining

$$(u_h^*, u_l^*) = \{log(a_h), log(a_l)\} \quad (9)$$

Now,

$$max_{u_h,u_l}(u_h - u_h^*)(u_l - u_l^*) \quad (10)$$

Then the solution goes in the favor of other SUs and they bargains for the first chance to get the sub-carrier. Desperate SUs have to support it for fulfilling their pay-off function.

Selfishness of the SU

$$\lambda = f\{T_b, q, n_l, n_{tl}\} \quad (11)$$

Here, $q = l - n_t$ represents the total number of sub-carriers left for auction and n_{tl} represents the number of turn at which SU got the sub-carrier within the group.

B. Dominant strategy solution (DSS) technique

In game theory, the concept of dominant strategy arrives when one strategy is better than another strategy for one player. It does not matter how that player's opponents may play.
For n-player game, each user has its dominant strategy and all users are playing *Bayesian games* [14]. Then the results for the dominant strategy will be:

- If any player's strategy dominates all other strategies then player with dominant strategy will lead the game.
- If none of the player's strategy dominates all other strategies then game finds an equilibrium point which is NE, also called as DSS.

Definition: For a selection set of strategy $\{(\alpha_i^*)_{t_i}\}_{i=1}^{n}$ (Where, $\{\alpha_k\}_{t_k}$ is the strategy played by A_k, with type t_k), is the dominant strategy. If for each type t_k of player A_k, for any $\{(\alpha_k)_{t_k}\}$ and any $\{(\alpha_{-k})_{t_{-k}}\}$ (t_{-k} represents all the type except k) for all the types t_{-k} of other players:

$$A_k\{t_k, \{\alpha_k^*\}_{t_k}, t_{-k}, \{\alpha_{-k}\}_{t_{-k}}\} \geq$$
$$A_k\{t_k, \{\alpha_k\}_{t_k}, t_{-k}, \{\alpha_{-k}\}_{t_{-k}}\} \quad (12)$$

In other words A_k needs the $\{(\alpha_k^*)_{t_k}\}$ strategy to get the sub-carrier. Consequently this strategy is played by A_k rationally. While, other may behave irrationally [9].

For the auctioning technique, sub-carrier is the pay-off of the SUs. The strategy for the SUs is to improve the bid $w.r.t$ (4), to get the sub-carrier. A_k always gets the sub-carrier, until A_k leaves the auction. This strategy yields the same result for each auction (if all players play rationally). Here, the selfishness λ of the SUs depend on the value of bid T_j^b.

$$\lambda = f\{T_j^b, l\} \quad (13)$$

C. Iterated Prisoner's dilemma (IPD) technique

To remove the dominance of DSS, IPD is used (here, all players will not play rationally. Some players may behave irrationally). This auction method will work on the principle of "prisoner's dilemma" [20]. In this strategy, SUs have two choices either to tell truth or to tell lie. Here, we have discussed why users will lie, what are the profits, what are the losses, and how our approach overcomes these issues.

- If SU tells a lie for reliability asset R_j^s:
  - To get the sub-carrier, R_j^s should be high.
    - * If SU does not fulfill the R_j^s provided, time taken by the SU to transmit data increases. After t duration, PUAP re-allocates that sub-carrier to somebody else. So, transmission of SU remains incomplete.
    - * To maintain the R_j^s, SU has to increase the rate by increasing some parameters. Considered parameter in this paper is power.
  - Increased power increase the probability of interfering with POB, because SUs don't know the limits. So, the R_j^{sp} is set to be 0.

TABLE I
tit for tat APPROACH [21].

Moves on Preceding rounds		
Player	Opponent	Suggested move
L(lie)	L	L
T(truth)	L	L
L(lie)	T	T
T(truth)	T	T

- If secondary user tells a lie for data D:
  - SU shows bigger chunk of data to increase the transmission time.
  - The value of reliability asset R_j^s depends on the data. Relation shown in (2).
- If SU is vulnerable for price Pr:
  - Price Pr is on the second position after the R_j^s. But, for very high price, SU can get the sub-carrier.
  - The selfishness parameter λ of user increases with each effort. After paying very high price, λ plays big role for the information rate achieved by SU.

For all these cases, if any user is found to be vulnerable, treated as a defaulter by PUAP and will be debarred for the further games. If all the users tell the truth or play fair, user with highest bid gets the auction.

SUs are the selfish users and they always tries to increase their pay-off function. To maintain the system stable model has to find the equilibrium conditions. The equilibrium conditions for SUs are:

- SUs have higher benefit, if they go for wrong ways else they have to wait. So, the equilibrium holds for first auction at "all users will tell lie".
  proof: Let u_1,u_2,u_3,...........,u_n are pay-off functions of the users, s_1,s_2,s_3,............,s_n are the strategies adopted by the SUs.
  If equilibrium is to be held for all the users
  $s_n \rightarrow s$ and $u_n \rightarrow u$
  and s_n counters u_n, so s counters u [13].
  For this game approach, strategy $s_n \rightarrow s$ is only to get the maximum pay-off i.e. $u_n \rightarrow u$ which is possible when user plays defectively.
- To find the optimum results, game should be iterated [15]. Here, *tit for tat* approach is used to implement IPD and solution depends on the previous decisions of the users [21] shown in *Table-I*.
- For two users, *tit for tat* approach yields efficient results. For multiple users *tit for tat* approach is found to be inefficient [12].

Chances of increasing the pay-off at first iteration is very low for the SUs. As the number of iterations increases, SUs can increase their pay-off functions reasonably. We have proposed a generalized form of *tit for tat* approach which is capable of handling multiple users efficiently.
The algorithm is as follows:

- For first auction, j^{th} SU tells the truth and plays rationally game with strategy $\{\alpha_j\}$.
  - A_j notes down the winning strategy $\{\alpha_k\}$ $(j \neq k)$.
  - Rationality of A_k with strategy $\{\alpha_k\}$ is found out using R_j^{sp}
- If $R_j^{sp} = 0$ for A_k, A_j will increment into the number of lies X_l
- If $R_j^{sp} = 1$ for A_k, A_j will increment into the number of truths X_t
- M_l = $mean\{X_l\}$
  M_t = $mean\{X_t\}$ (Here, M_l, M_t are the mean of telling the truth and mean of telling a lie respectively.)
- If $M_l > M_t$
  Strategy followed by A_j is $\{\alpha_j\}$
  Else
  Strategy followed by A_j is $\{\alpha_j'\}$ (represent the irrational strategy of A_j)

This algorithm is the generalized form of *tit for tat* algorithm involving multiple users. After few iteration, SUs will get the fairly possible pay-off function using this algorithm. Here, the selfishness λ (corresponding to IPD) of the user is defined as,

$$\lambda = f\{T_j^b, n_t, l\} \tag{14}$$

Where, n_t represents the total number of iterations after which SUs gets the sub-carrier.

Now, the effect of the approach proposed by us is:

- Game strategy by A_j:
  For the first iteration, A_j plays fair and tells the truth.
  - If all other players play fair:
    For remaining all strategies, dominant strategy solution will be adopted and the player having dominant strategy get the sub-carrier (here, strategy is meant for proposed bid).
  - If all players tell a lie:
    For this condition again, the dominant strategy solution comes into existence, but player playing fair have a chances to lose the bid.
  - If maximum players play fair:
    For this condition, players playing defectively get the sub-carrier, remaining players loose the chance.
    The A_j following the proposed approach, notes down the winning bid and check its authenticity by the help of PUAP. (PUAP forwards the bid to next users if the winner is found to be defective using the Markov chain process see Fig. 2):
    The A_j finds out the mean of playing defect by the winners M_d and also the mean of playing fair by the winners M_c. Now, the decision of the A_j for next auction is:
    * M_c>M_d:
      A_j will play fair in next auction.
    * M_d>M_c:
      A_j will play defectively in next auction.
    * M_c=M_d:
      A_j will play fair in next auction.
- Equilibrium condition for first game is same as for DSS: NE holds for the game when all the players play their dominant strategy [14].
- Equilibrium condition for further iterations:
  Iteration removes the dominance of the dominant strategy

solution, but condition is that all players should not be rational player; otherwise result will always be DSS.

IV. Proposed Hybrid approach for Spectrum leasing

We have discussed about the different strategies adopted by SUs to get the auction and noticed that each SU will try to increase its pay-off function (sub-carrier for the auction). If SU struggles to acquire the sub-carrier then SU have higher value of λ. SU with high value of λ can cross the POB and provides a non-tolerable interference to the other users [6].

SUs use the digital beamforming (DBF) techniques to transmit their data. Maximum-ratio transmission (MRT), zero-forcing (ZF) beamformers are well known in literature. The MRT beamforming vectors maximize the power of the received desired signal component. The ZF beamformers assure that the transmitter generates no interference to the other transmitting nodes and the detailed description of both the type of beamforming with the effect on POB is given in [22]

$$w(\lambda_i) = \frac{|\ (1-\lambda_i)w_z + (\lambda_i)w_m)\ |}{\|\ (1-\lambda_i)w_z + (\lambda_i)w_m)\ \|} \tag{15}$$

where, w_z, w_m reprsent the ZF and MRT beamforming respectively and i = 1,2,.....,n are the number of users. The type of DBF technique adopted by SUs depends entirely on λ [23].

- For $\lambda = 1$:
  SU plays purely selfish game and goes for MRT beamforming.
- For $\lambda = 0$:
  SU is a non-selfish player, opts for ZF beamforming technique.
- For $0 < \lambda < 1$:
  SU uses combinational technique of MRT and ZF. Effect of ZF increases in combination if $\lambda \to 0$ and vice versa for MRT.

For e.g, the information rate r achieved by the SU1 will be:

$$r = log_2(1 + \frac{|\ w(\lambda_1)h_{11}\ |^2}{\|\ w(\lambda_2)h_{21}{}^2 + \sigma^2\ \|}) \tag{16}$$

Here, h_{11} represents the channel gain from transmitter SU1 to receiver AP1 (access point), h_{11} represents the interference from SU2 to AP1, and r is the pay-off function of SUs after getting sub-carrier.

The algorithm followed by PUAP to assess the reliability R_j^{sp} is given below.

- t is the total time needed by SU for transmission. T is the time assumed for probation period and $T > 0$. During this period, PUAP will analyze the pay-off function $u(k)$ of k^{th} SU.
  - This process will repeat till $T = 0$
    PUAP considers $R_j^{sp} = x$ (where, $x = 1$ initially).
    As $u(k) = f\{\lambda\}$ (Here, $u(k)$ is for the information rate and the relation is from, Eqn. (16))
  - If $u(k) > P_a$ (Here, P_a represents the POB):
    There is change in the value of x now
    $x = (x - \frac{1}{T})$
    So, the new value is:
    $R_j^{sp} = x$
  - If $u(k) < P_a$:
    There is no change in the value of R_j^{sp}
    So, the value is:
    $R_j^{sp} = x$
- Finally, PUAP checks the value of R_j^{sp}:
  If $R_j^{sp} < \gamma$ (γ represents threshold limit)
  $R_j^{sp} = 0$ (user marked as non-reliable)
  Else
  R_j^{sp}=1 (user marked as reliable)

Threshold value is to be set by PUAP depending on the channel matrix and number of SUs transmitting simultaneously.

A. SU playing non-cooperative game with PUAP

SU starts transmitting over the allocated sub-carrier independently. During the probation period, PUAP will analyze the pay-off of the SU.
Then the game formation will be:

$$G = \{[A_k, PU_{AP}], [\alpha(k), s(p)], [u(k), u(p)]\} \tag{17}$$

Here, $s(k)$, $s(p)$ represents the strategy adopted by the SU and strategy adopted by PUAP (PU_{AP}) respectively. Pay-off function $u(k)$, $u(p)$ is the measures of achieved rate by SU and PU respectively.

For probation period, $\alpha(k)$ is independent of $s(p)$. Reliability asset of SU R_j^{sp} is set by PUAP and discussed earlier. R_j^{sp} depends on the pay-off of the SU $w.r.t$ to the POB. SUs violating the POB produce an intolerable interference to the PU. Whenever the interference caused by the SUs exceed the POB, PUAP will degrade the value of R_j^{sp} from default value one. PUAP sets the value of $R_j^{sp} = 0$, if it is found below a threshold value. PUAP extends the probation period and set $R_j^{sp} = 1$, if it is found above threshold value.

B. SU playing cooperative game with PUAP

The SUs having R_j^{sp}=1 are marked as reliable users. Game strategy followed by PUAP changes from non-cooperative to cooperative. PUAP will share the information i.

$$i = \{C_{o,p}, E\{|h_p|^2\}, p_p\} \tag{18}$$

Here $C_{o,p}$ represents the target transmission rate of the PU, $E\{|h_p|^2\}$ represents the average channel gain of primary channel, and p_p represents the power transmitted by PU. The information i will allow SU to maintain power transmission p_s within limits. For cooperative game the strategy $\alpha(k)$ is dependent on strategy $s(p)$.

If P is the maximum power transmitted by the SU without crossing POB, then $p_s \leq P$. Selfishness of SU changes *w.r.t* to the i shared by PUAP.

$$\lambda' = f\{i, \lambda\} \tag{19}$$

Then the value of pay-off function is:

$$R = log_2(1 + \frac{|\ w(\lambda_1')h_{11}\ |^2}{\|\ w(\lambda_2')h_{21}{}^2 + \sigma^2\ \|}) \tag{20}$$

The pay-off function of SU is the information rate $R \geq r$, which depends on the power transmitted p_s. It may be noted that, R is the information rate for cooperative strategy and r is for non-cooperative strategy. Here, the power transmitted by the SU is the important part and the strategy adopted by the SU is to maximize its power transmission in order to increase its pay-off, without exceeding the POB.

1) Profit of playing cooperative game:

- Profit of SU:
  - SU's pay-off is increasing with the reasonable amount.
  - Total transmission time for the SU reduces.
- Profit of PUAP:
  - Total transmission time for sub-carrier reduces *i.e.* it becomes free sooner.
  - Respect and authenticity of PUAP increases and attract more users for next auction.

Bargaining problem for this approach is; SUs and PUAP both are benefited, if they play cooperative game. We have formulated a bargaining problem between SUs and PUAP. Here, pay-off of both SUs and PUAP with bargaining [19] are:

$$u_p^{bargain} = log(r_p + R_p) \quad (21)$$

Here, r_p, R_p is the pay-off of PUAP for non-cooperative play and cooperative play respectively.

$$u_s^{bargain} = log(r_s + R_s) \quad (22)$$

Here, r_s, R_s is the pay-off of SU for non-cooperative play and cooperative play respectively. For the case for no bargaining:

$$(u_s^*, u_p^*) = \{log(r_s), log(r_p)\} \quad (23)$$

Now,

$$max_{u_h,u_l}(u_s - u_s^*)(u_p - u_p^*) \quad (24)$$

From the above equations, the profit obtained by both SUs and PUAP are equal. So there is no need of bargaining for this case or in other words both the players are equally benefited from this collation.

V. Algorithm to re-allocate the empty sub-carriers

PUAP releases one sub-carrier in each auction and also calculates the maximum time of transmission t for each sub-carrier. After t duration, PUAP re-allocates the sub-carrier to other SU without any charge. This re-allocation of sub-carrier reduces the time of transmission for the SU by the significant factor.

This process reduces the total time of transmission and makes it just half. PUAP gets spectrum earlier. So PUAP can easily re-auction the spectrum.

Approach for reducing transmission time:

- Calculation of estimated time of transmission t.
  PUAP estimates the total time of transmission on the basis of bid provided by SU and calculates the estimated time t for each sub-carrier.
- Sub-carrier reallocated after t duration.
  PUAP reallocates the sub-carrier to other SU which is having maximum value of D to be transmitted.
- Total time of transmission reduces by half.

Let, PUAP1 uses this approach for reducing transmission time. Here, Stackelberg's competition model is used. PUAP1 will increase the number of SUs in the next auction with the help of Stackelberg's competition model. PUAP1 get the maximum benefit from the auction, if the value of n increases. PUAP1 gets whole spectrum earlier for the next auction. The number of sub-carriers of PUAP1 is more compared to the sub-carriers of other PUAPs.

Definition : For the competition model between two PUAPs. $l_1 + \bar{l_2}$ is the total sub-carriers available in the market for auction ($\bar{l_2}$ is the number of sub-carriers of PUAP2, after first auction of spectrum, initially it was l).

Number of SUs depends on the $a - b(l_1 + (\bar{l_2}))$. Here, $a > 0$, $b > 0$ are the constants and parameters of the demand curve [14].

Total profit obtained by PUAPs (gaining maximum number of SUs) is $[a - b(l_1 + \bar{l_2})]l_1 - cl_1$. Here, c is the marginal cost of sub-carriers payed by PUAPs to relevant authorities. Following equation defines the profit gained by PUAPs is:

$$max^{l_1}[a - b(l_1 + \bar{l_2}) - c]l_1 \quad (25)$$

Now, the best response (BR) of the PUAPs is.

$$BR_1(l_2) = \begin{cases} \frac{a-c-b\bar{l_2}}{3b} & \text{if } l_2 \leq \frac{a-c}{b} \\ 0 & \text{if } l_2 > \frac{a-c}{b} \end{cases} \quad (26)$$

$$BR_2(l_1) = \begin{cases} \frac{a-c-bl_1}{3b} & \text{if } l_1 \leq \frac{a-c}{b} \\ 0 & \text{if } l_1 > \frac{a-c}{b} \end{cases} \quad (27)$$

$l_1 > \bar{l_2}$, PUAP1 uses the algorithm to re-allocate the sub-carriers. For Stackelberg model PUAP1 will assumes that the $BR_2(l_1)$ is choice of the PUAP2 for any value of l_1. So, that PUAP1 will solve this problem to get maximum benefit [14].

$$max^{l_1}[a - b[l_1 + BR_2(l_1)] - c]l_1 \quad (28)$$

Solving the above equation result is:

$$max^{l_1}\frac{1}{2}[a - bl_1 - c]l_1 \quad (29)$$

From the above equations, PUAP1 force the PUAP2 to cut back its sub-carriers for auction and PUAP1 gains maximum number of SUs.

VI. Numerical Results

In this section, we illustrate the transmission behavior of the SUs with the help of numerical results. We consider a simple geometrical model where SUs are placed at approximately same distance d $(0 < d < 1)$ from PUAP and $1 - d$ from primary receiver. Considering the channel as Rayleigh fading channel, we assume average channel gain of primary channel is 1, $E[|h_p|^2] = 1$ and average channel gain $E[|h_{p,s}|^2] = 1/d^n$, $E[|h_{s,p}|^2] = 1/(1 - d^n)$, and $E[|h_s|^2] = 0.8$ for PU to

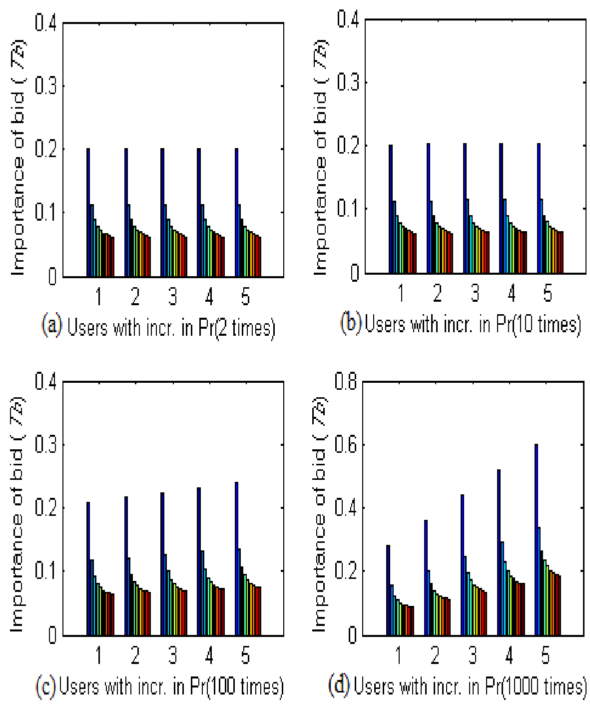

Fig. 5. Simulation result showing the variation in the values of normalized T_b, $w.r.t$ to the R_j^s which varies 0 to 1 for each user. Pr is varying from user to user with a specific rate.

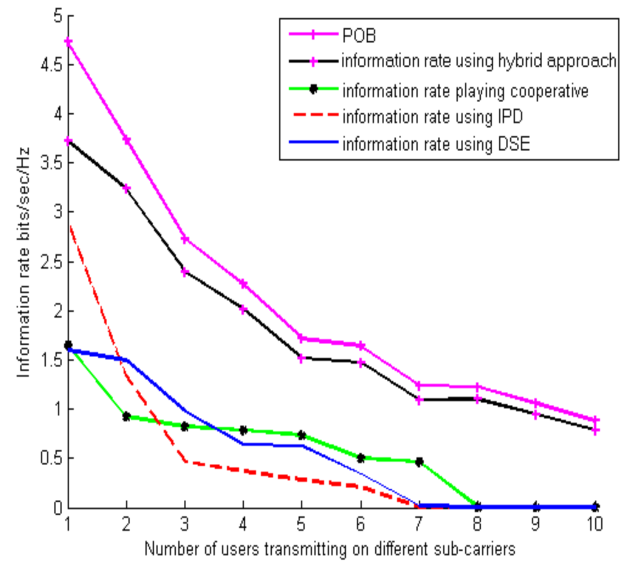

Fig. 6. Comparison between DSS, IPD, Cooperative strategy with proposed hybrid approach $w.r.t$ POB

SU, SU to PU and for secondary channel respectively [17]. Pr is the price proposed to the PUAP by SU, depends on l. We further assume $Pr > l^2$ as a initial condition for the auction. We have applied ZF beamforming for the rational users. MRT beamforming is used by irrational users. Here, we consider $l = 10$ the number of sub-carriers allocated to SUs.

In Fig. 5, succeeding users have increment in the value of Pr of 2, 10, 100, and 1000 in fig. 5-(a), 5-(b), 5-(c), and 5-(d) respectively. Further each users have a value of T_j^b for the values of R_j^{sp} varies from 1 with decrement of 0.1 till it reaches to 0. Firstly T_j^b is calculated as follows:

$$T_j^b = \frac{Pr}{\{1 - R_j^s\}^2 l} \tag{30}$$

Here, R_j^s effect the value of T_b maximum shown in Fig. 5. Effect of Pr depends on the value of l. For small values of l, Pr is dominant parameter. The selfishness factor λ depends on the strategy adopted by the SUs to acquire the sub-carriers from PUAP. Selfishness for the SUs opting for DSE strategy, IPD strategy and cooperative strategy depends on the parameters given in Eqn. (13), Eqn. (14) and Eqn. (11) respectively. λ used in numerical calculation, for each strategy is given below:

- Selfishness for the cooperative strategy varies inversely with the value of q. λ is directly proportional to the value of number of users left for sub-carrier ($\|T_b\|$ is the normalized form of T_b).

$$\lambda = \frac{\|T_j^b\|(n_l - n_{tl})}{q} \tag{31}$$

  Here, n_{tl} represents the turn at which SUs get the sub-carrier within the group, $n_{tl} \leq n_l$. λ achieves the maximum peak 1 for $(n_l - n_{tl}) = q$ and $T_b = 1$ i.e, SU showing having $max(T_b)$ but due to bargaining condition turn of SUs delayed by n_{tl}.
- Selfishness for the DSS strategy varies inversely with value of l.

$$\lambda = \frac{\|T_j^b\|}{l} \tag{32}$$

  λ increases if $l \to 0$ (for $l = 0$ there is no auction.). λ of each user depends on its dominant strategy.
- Selfishness for the IPD strategy is directly proportional to the number of iterations.

$$\lambda = \frac{\|T_j^b\| n_t}{l} \tag{33}$$

  λ of IPD strategy is dependent on other strategies, adopted by other players.

POB is the important parameter for calculating the value of R_j^{sp}. If there are n users transmitting simultaneously, achievable rate region for each user can be easily calculated. The outer boundary of the total achievable rate region is POB [22].

A. comparison of proposed hybrid approach with other strategies

In this section we compared our proposed hybrid approach with the previous hybrid approach as well as other game strategies.

TABLE II
COMPARISON OF COMPLEXITY BETWEEN DIFFERENT STRATEGIES

Strategy→, Complexity ↓	Cooperative	DSS	IPD	GMBR (hybrid users) [8]	Proposed hybrid
Space	n_l	1	$1 + n_t$	$1 + n$	$3 + n$
Time	n	1	$1 + 3n_t$	$2n + n_f$(n_f is the number of users not stable and repeats the process again)	$3T + 1$

1) comparison with previously adopted hybrid approaches: There are two types of hybrid approaches adopted earlier for spectrum sharing:

- Hybrid users [24]:
  In this approach, users are hybrid $i.e.$ the number of SUs and PUs are variable. PUs shares the spectrum with SUs but does not have any feedback mechanism.
- Hybrid users with incentives, formulation of Game Model Based on Reputation (GMBR) [8]:
  GMBR uses the incentive technique with hybrid game (users are variable). SUs use the incentive technique as feedback for a PU and calculate its effectiveness. This approach is SU oriented and formulate the game provider vs provider [25].
- Proposed hybrid approach:
  Our proposed model is a type of GMBR. Here, strategy is hybrid instead of users. PUAP controls the model and dynamically change strategy, according to the behavior of SU. PUAP limits the interference provided by SUs within the POB. R_j^{sp} is used as the feedback which makes this strategy more robust and effective compared to others. The complexity comparison of the proposed hybrid approach shown in Table II:
  - Space complexity of the proposed approach is higher then all other strategies. The complexity of the proposed approach is for PUAP while complexity of other strategies is for PUs. For large values of n complexity of GMBR (hybrid user) and our approach are appeared to be same.
  - Time complexity of the proposed approach depends on the value of probation period T. For T $\rightarrow$ 0 value of time complexity also decreases. Whereas, complexity of GMBR (hybrid users) depends on the value of n and n_f.

  World is made up-off trade -offs, if we gain something we has to lose something. The proposed strategy limits the interference caused by SUs to PUs at the cost of slight increase in the system complexity.

2) comparison with other game strategies: In Fig. 6, we have plotted information rate of SUs using different game strategies including DSS, IPD, cooperative, and hybrid *w.r.t* POB. It may be noted that, there is the improvement in the average rate achieved by SUs, with hybrid approach. As the number of SUs increases, results found to be deviating towards lower limit. Those SUs, opted for cooperative strategy, transmit their data on fairly well rate compared to other strategies. Here, SUs opting for cooperative strategy means for the collation between the transmitting SUs.

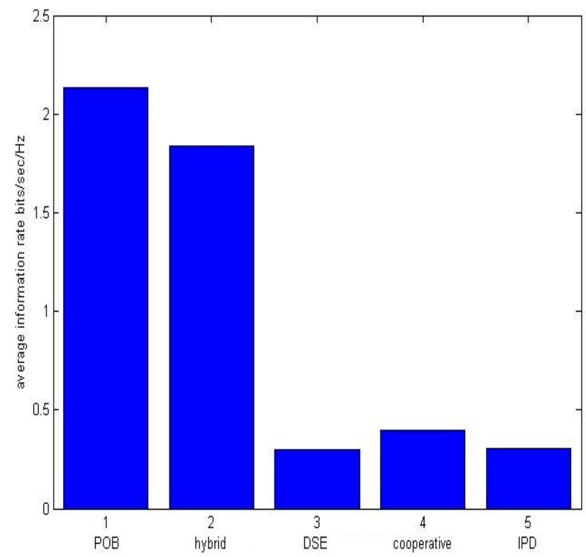

Fig. 7. Average information rate achieved by users using hybrid, DSS, Cooperative, and IPD technique $w.r.t$ POB.

Results shown in Fig. 6 demonstrates that the SUs, acquiring sub-carriers initially, go for ZF beamforming. Selfishness of the SUs depends on the number of retries. As SUs do not get sub-carriers, λ will keep on increasing. Those SUs adopt MRT beamforming having higher value of λ. We also noted that the information rate increases by a significant margin if we implement the hybrid approach as compared to other approches shown in Fig. 7.

B. re-allocation of empty sub-carriers

PUAP reduces the transmission time for the SUs. PUAP reallocate the sub-carrier to another SUs having higher value of D, after t duration. This model reduces the overall transmission time for the SUs over spectrum by half of the original time as shown in Fig. 8. PUAPs can re-auction the spectrum using three different techniques.

- PUAP1:
  Follows the re-allocation technique we have discussed in section IV.
- PUAP2:

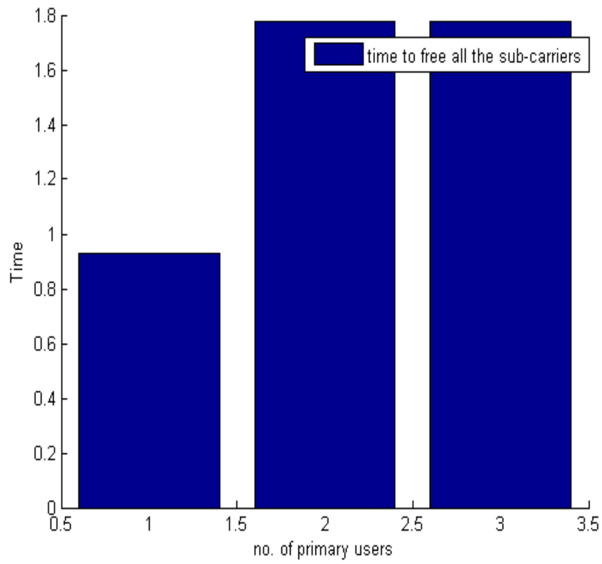

Fig. 8. Simulation results showing the SUs average time of transmission $w.r.t$ PUAP1, PUAP2, and PUAP3.

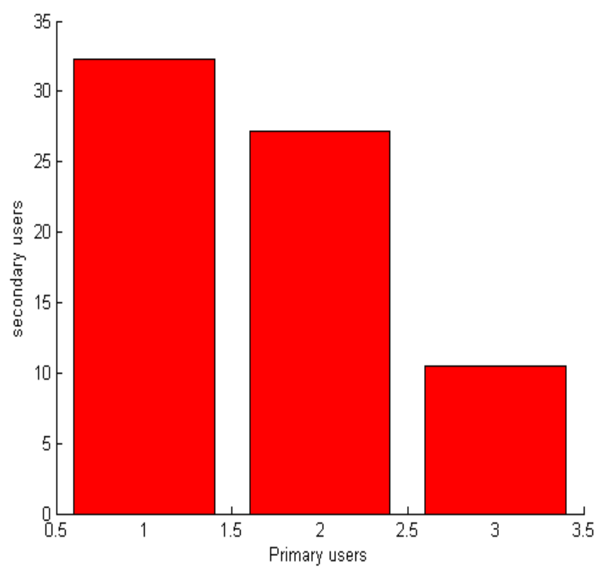

Fig. 9. Simulation results showing the SUs gained for re-auctioning by PUAP.

Wait for all the SUs to finish transmission and then re-auction the whole spectrum again.

- PUAP3:
  Auctions the sub-carriers as soon as they released.

Fig. 9 gives a statistical view for the number of SUs participating in different auctions. With the help of Stackelberg's competition model, PUAP1 managed to gain the maximum number of SUs compared to other PUAPs. PUAP2 is also re-auctioning the whole spectrum but it waits for the long time as shown in Fig.8. PUAP2 manages to be second-in-command after PUAP1.

VII. Conclusion and Future Work

In this paper we have implemented a hybrid model for spectrum sharing in CRN. Here, PUAP controls the spectrum leasing process and use adaptive technique to change strategies dynamically. SU plays to maximize their utility, PUAP plays according to SUs and maintain the level of interference below the POB. It has been shown that hybrid strategy improves the efficiency of the system, by increasing the average information rate of the users. Pay-off function obtained by the SUs is maximized without violating other user's interest. PUAP re-allocates the empty sub-carrier to other SU and considerably reduces total transmission time of SUs. PUAP also implements the Stackelberg's competition model to increase the number of participating SUs in next round which increases the payoff of PUAP. In this paper application of Markov chain model is proposed to remove the vulnerability of Vickery auction mechanism.

In future work, reliability of the SUs which were marked as non-reliable in the first round of auctioning would be considered for further rounds of auctioning.

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